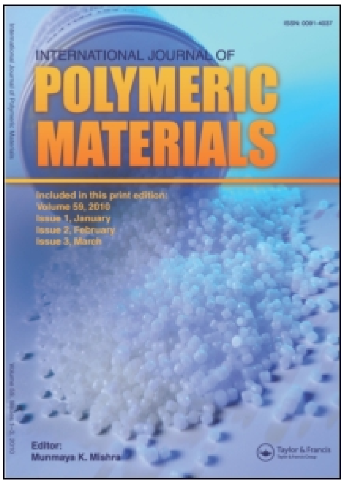

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A Simple Dilatometric Method for Determining Poisson's Ratio of Nearly Incompressible Elastomers

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A simple apparatus which was developed for measuring the dilatation of specimens tested in uniaxial tension is described. The dilatometer can be used on an Instron testing machine. In spite of its simplicity, this dilatometer enables an accurate determination of Poisson's ratio of nearly incompressible elastomers. We present typical curves showing the effect of strain on Poisson's ratio of filled and unfilled elastomers. We also describe the dilatometric processes observed during straining of granular filled elastomers.

INTRODUCTION

Poisson's ratio, ν, is one of the fundamental properties of materials. While for any amorphous or polycrystalline linearly elastic material Poisson's ratio is constant, for viscoelastic materials it is a function of time as well as of strain.[1] Due to dewetting processes occurring during the straining of elastomers filled with a granular material, an additional effect on Poisson's ratio is observed.[2]

In order to simplify engineering problems dealing with viscoelastic materials, approximate elastic behaviour with a constant Poisson's ratio is frequently assumed. The solutions to such problems often include a factor $(1 - 2\nu)$. In the case of nearly incompressible elastomers, for which Poisson's ratio is nearly 0.5, the factor $(1 - 2\nu)$ and particularly its reciprocal may have a considerable influence on the calculations, and small differences in the assumed value of ν may lead to completely invalid results.

There are several methods which may be used for the determination of Poisson's ratio:

a) An indirect method in which two moduli of the material are determined experimentally and Poisson's ratio is calculated by means of the well-known elastic equations.[3]

b) Methods in which lateral and longitudinal strains are measured simultaneously during extension or compression by means of strain-gages, extensometers or optical techniques.

c) Dilatometric methods in which changes in volume are measured while the specimen is strained. Poisson's ratio is then calculated by means of its relation to dilatation and strain.

In the case of soft almost incompressible rubbery materials, the dilatometric method is preferable. The value of Poisson's ratio as determined by the dilatometric method is the same as would have been obtained by averaging the values from measurements at each point of the specimen. This feature of the dilatometric method is of great importance in the case of granular filled elastomers, since measuring their deformations at a given point may lead to erroneous results due to the local non-uniform nature of these materials.

Because of these advantages we decided to adopt the dilatometric method for the determination of Poisson's ratio of our elastomers.

In the literature several dilatometers are described, most of them were built for measuring dilatation of rubber rings.[4,5] For uniaxial extension a magnet was used,[6] but this method can be applied only for small deformations of very soft materials. Farris[2] built a gas dilatometer which can measure accurately the dilatation occurring while filled and unfilled elastometric specimens are extended. But in order to achieve the required accuracy, when a gas is used as the measuring media, very expensive control systems are required.

We designed and built a simple apparatus for room temperature measurements capable of installation in an ordinary Instron testing machine. We used water as a measuring media, therefore our dilatometer does not require special expensive additional equipment.

In spite of its simplicity, our dilatometer, which is described later, enables the accurate determination of Poisson's ratio of nearly incompressible elastomers as a function of strain and rate of strain. This dilatometer also enables observation of dewetting processes occurring in granular filled elastomers. If the friction in the extension mechanism is compensated, stress can also be determined simultaneously to dilatation measurement so that the bulk modulus can be calculated.

DEFINITIONS OF POISSON'S RATIO

In a rod under uniaxial stress, Poisson's ratio is defined as minus the ratio of lateral strain ϵ_y to the longitudinal strain ϵ_x:

$$\nu = -\epsilon_y/\epsilon_x \tag{1}$$

As a result of the various definitions of strain, a number of expressions for Poisson's ratio are obtained:

a) For small deformations, the Cauchy measure of strain is usually used, for an extension ratio λ the strain ϵ equals:

$$\epsilon = \lambda - 1 = l/l_0 - 1 = \Delta l/l_0 \tag{2}$$

where l_0 and l are the initial and strained lengths respectively.

b) For large deformations, a logarithmic definition of strain is used:

$$\epsilon = ln\,\lambda \tag{3}$$

In this case, a logarithmic Poisson's ratio is obtained, known as Hencky's definition:

$$\nu = -ln\,\lambda_y/ln\,\lambda_x \tag{4}$$

The above expressions for Poisson's ratio are sometimes generalized by using differential such as:

$$\text{Differential Cauchy:}\quad \nu = -d\epsilon_y/d\epsilon_x \tag{5}$$

$$\text{Differential Hencky:}\quad \nu = -d\,ln\,\lambda_y/d\,ln\,\lambda_x \tag{6}$$

These differential expressions are less sensitive to extension than the respective undifferentiated expressions for Poisson's ratio.

For elastic materials which obey Hooke's law, Poisson's ratio is related to the Young and Bulk moduli, E & K, by the well known relation:

$$\nu = \tfrac{1}{2}\left(1 - \frac{E}{3K}\right) \tag{7}$$

While there are additional definitions of Poisson's ratio,[7] we shall restrict our attention to those given and later interpret our experimental results according to the classical definitions (1), (4) and (7), which relate to a wide range of deformations.

THE RELATIONS BETWEEN DILATATION AND POISSON'S RATIO

We consider a uniform isotropic square sectional specimen whose initial volume is: $V_0 = xy^2$. When the specimen is strained in the longitudinal direction, its volume increases and becomes:

$$V = V_0 + \Delta V = (x + \Delta x)(y + \Delta y)^2 \tag{8}$$

The relative dilatation is given by the following:

$$\frac{\Delta V}{V_0} = \frac{\Delta x}{x} + 2\frac{\Delta y}{y} + \frac{\Delta y^2}{y^2} + \frac{2\Delta x \, . \, \Delta y}{xy} + \frac{\Delta x \, . \, \Delta y^2}{xy^2} \tag{9}$$

If, in Eq. (9), we substitute the strains as defined earlier, this substitution will lead to various relations between the dilatation and Poisson's ratio. For instance, for Cauchy's definition of strain, the following expression is obtained:

$$\nu = \frac{1}{\epsilon_x}[1 - \left(\frac{V_0 + \Delta V}{V_0}\right)^{1/2} (1 + \epsilon_x)^{-1/2}] \tag{10}$$

In the case of infinitesimal deformations, when second and higher order terms of deformation are neglected, the relative dilatation equals:

$$\frac{\Delta V}{V_0} = \frac{\Delta x}{x} + 2\frac{\Delta y}{y} \tag{11}$$

Hence, for Cauchy's strain, Poisson's ratio is:

$$\nu = \frac{1}{2}\left[1 - \frac{1}{\epsilon_x}\left(\frac{\Delta V}{V_0}\right)\right] \tag{12}$$

We would like to point out the fact that the same expression is obtained if, in Eq. (7), we substitute the relation between the bulk modulus and the dilatation for elastic materials:

$$K = \frac{\sigma}{3} \Big/ \left(\frac{\Delta V}{V_0}\right) \tag{13}$$

and applying Hooke's law for the stress $\sigma = E\epsilon_x$.

A summary of the various definitions of strain, the resulting expression for Poisson's ratio and their relation to dilatation is given in Table I. From these relations two important points should be noted:

a) For all definitions of strain, Poisson's ratio is generally strain dependent.

b) However, for incompressible materials, i.e. $\Delta V = 0$, Poisson's ratio changes with strain only for Cauchy's definition. For Hencky's and the Engineering definitions, Poisson's ratio is constant and equals $\frac{1}{2}$.

TABLE I

The relations between dilatation and Poisson's ratio, according to the various definitions of strain

	Definition	Strain elongation relation	Poisson's ratio	Dilatation—poisson's Ratio relation	
				Compressible material	Incompressible material
1	Cauchy	$\epsilon = \lambda - 1$	$\nu = -\frac{\epsilon_y}{\epsilon_x}$	$\nu = \frac{1}{\epsilon_x}\left[1 - \left(\frac{V_0 + \Delta V}{V_0}\right)^{1/2}(1+\epsilon_x)^{-1/2}\right]$	$\nu = \frac{1}{\epsilon_x}[1 - (1+\epsilon_x)^{-1/2}]$
2	Hencky	$\epsilon = ln\,\lambda$	$\nu = -\frac{ln\,\lambda_y}{ln\,\lambda_x}$	$\nu = \frac{1}{2}\left[1 - \frac{ln\left(\frac{V_0+\Delta V}{V_0}\right)}{ln\,(1+\epsilon_x)}\right]$	$\nu = \frac{1}{2}$
3	Differential Cauchy	$\epsilon = \lambda - 1$	$\nu = -\frac{d\epsilon_y}{d\epsilon_x}$	$\nu = \frac{1}{2}\left(\frac{V_0+\Delta V}{V_0}\right)^{1/2}(1+\epsilon_x)^{-3/2}$	$\nu = \frac{1}{2}(1+\epsilon_x)^{-3/2}$
4	Differential Hencky	$\epsilon = ln\,\lambda$	$\nu = -\frac{d\,ln\,\lambda_y}{d\,ln\,\lambda_x}$	$\nu = \frac{1}{2}\left[1 - \frac{d\,ln\left(\frac{V_0+\Delta V}{V_0}\right)}{d\,ln\,(1+\epsilon_x)}\right]$	$\nu = \frac{1}{2}$
5	Engineering Hooke's law	$\epsilon = \lambda - 1$	$\nu = \frac{1}{2} - \frac{E}{6K}$	$\nu = \frac{1}{2}\left[1 - \frac{1}{\epsilon_x}\frac{\Delta V}{V_0}\right]$	$\nu = \frac{1}{2}$

MATERIALS AND SPECIMEN PREPARATION

The materials tested were elastomers based on low molecular weight polybutadiene. Asbestos powder was used as a filler.

Since the expected changes in volume are very small and the accuracy required is high, a specimen with a large initial volume 25 × 25 × 130 mm was chosen. Because we intended to measure dilatation in a uniaxial stress field, we compared the stress–strain behaviour of this specimen with that of standard uniaxial specimens, and observed no significant difference.

The specimens were prepared by casting in a special Teflon-coated mold. The casting was at elevated temperature and at low pressure to avoid the formation of bubbles in the specimen.

Two metal terminal blocks (see Figure 1) were placed in position in the mold before casting, and these adhered to the specimens during curing.

EXPERIMENTAL APPARATUS AND TECHNIQUE

A dilatometer which could be installed on an Instron universal testing machine was built. A schematic representation of the dilatometer is shown in Figure 1 and a photograph is given in Figure 2.

The dilatometer chamber consists of a Pyrex tube closed by two metal caps coated with Teflon. The two caps are tightened by means of four studs. The tabs of the specimen are screwed to the upper cap and to a uniform diameter stainless steel pulling rod.

The upper cap of the dilatometer is connected to the load cell of the testing machine and the pulling rod is connected to the moving cross-head. The pulling rod, 2.1 mm in diameter, passes through the lower cap and is sealed by O-rings.

The changes in volume are measured by a graduated capillary tube, 1 mm in diameter, connected in parallel to the chamber.

The liquid, water with a few drops of ink to improve visibility, is introduced into the dilatometer through a lower cap. Removal of air and complete filling is ensured by means of a valve placed at the upper end.

The measurements were performed at room temperature. In our dilatometer small fluctuations in temperature will not affect the accuracy of dilatation measurements, as it is in the case of a gas dilatometer.

In order to check the influence of water on the materials tested, several specimens were immersed in water for 20–30 minutes (about twice the duration of a measurement). When compared to unimmersed specimen, no swelling and no significant changes in the mechanical properties were observed.

By inserting some mercury into the capillary tube and weighing different lengths of the mercury column at various locations, the uniformity of the

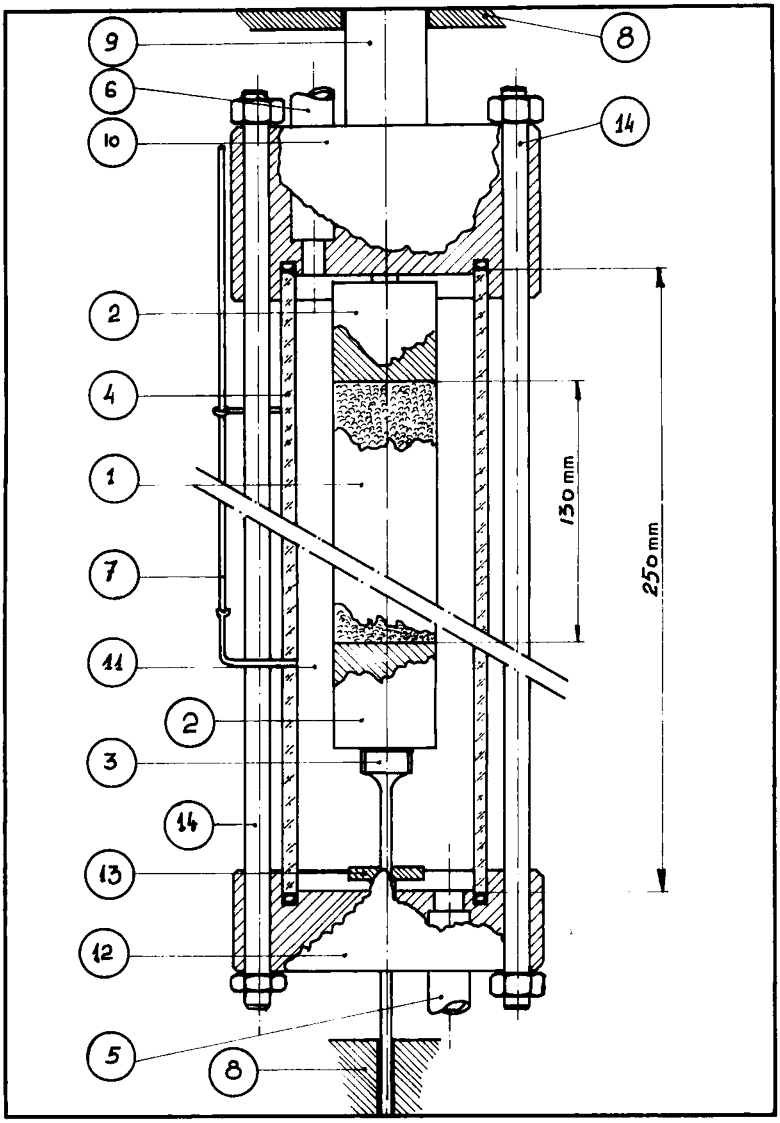

FIGURE 1 Schematic description of the dilatometer. 1. The specimen. 2. Metal terminal blocks. 3. Pull rod. 4. Pyrex glass tube. 5. Opening for introducing the liquid used as measuring media. 6. Opening for through-flow of liquid. 7. Graduated through bore capillary tube. 8. Jaws of the tension machine. 9. Connection piece. 10. Upper cap of the dilatometer. 11. The liquid. 12. Lower cap of the dilatometer. 13. Sealing of the pass of the pull rod. 14. Studs connecting the caps of the dilatometer.

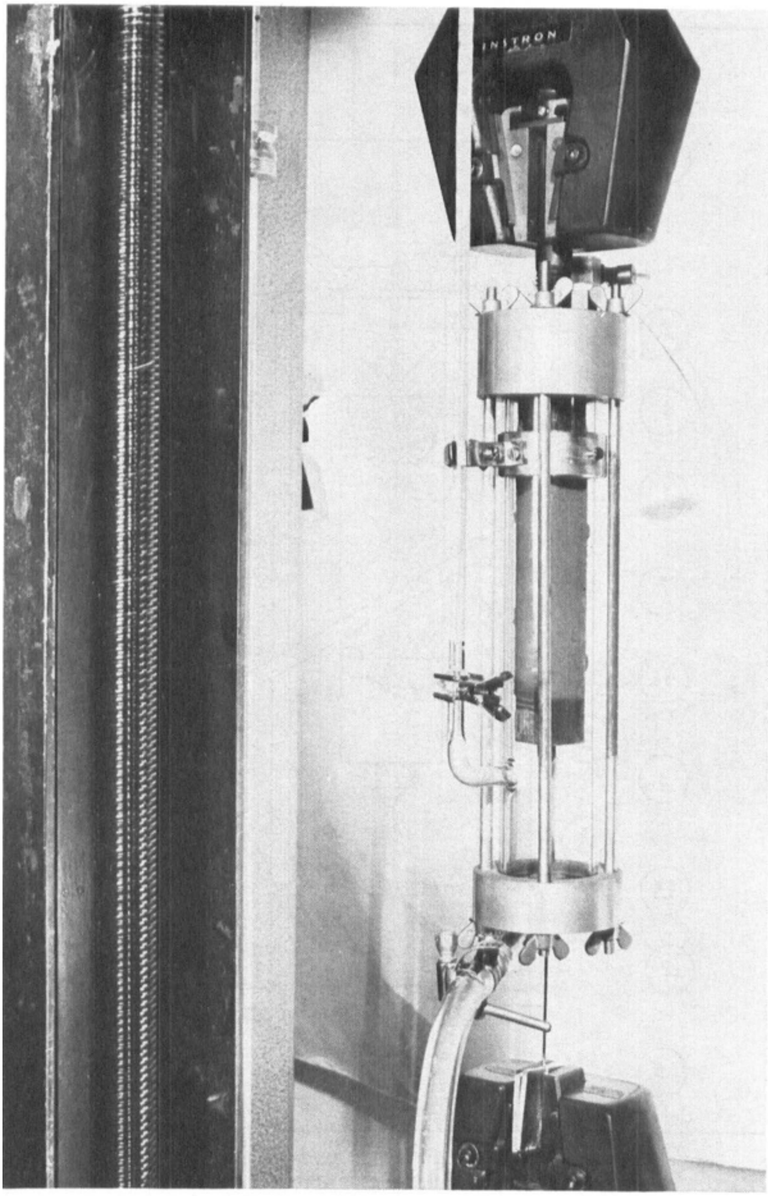

FIGURE 2 The dilatometer assembled on the Instron testing machine.

capillary tube was checked. The average volume per 1 cm length (v_c) was calculated from a calibration curve and by a least square technique

$$v_c = 6.2 \times 10^{-3}\ \text{cm}^3/\text{cm} \qquad d(v_c) = 6.2 \times 10^{-4}\ \text{cm}^3/\text{cm} \tag{14}$$

where $d(v_c)$ is the standard deviation.

A base line for the changes of volume of the specimen was obtained by performing a test with no specimen in the dilatometer. The movement of the meniscus in the capillary tube was found to be proportional to the length of rod pulled out of the chamber, following the relation

$$h_m = A\,h_c = 0.31\,h_c \pm 0.008[\text{cm}] \tag{15}$$

where h_m is the movement of the rod [cm]

A is the ratio of the volume per length of the capillary tube v_c and of the rod v_m; $A = v_c/v_m$.

The accuracy in reading the height of the liquid column in the capillary—h_c had a standard deviation of 0.05 cm.

The force necessary to overcome the friction caused by the O-rings sealing the pulling rod was found to be uniform and equals 200 grams, which is negligible in comparison to the force necessary to extend the specimens.

Because the rigidity of the steel pulling rod is higher than that of the materials investigated and has almost the same length, the deformation of the rod can be neglected so that all the measured extension is due to the strain in the specimen.

The initial volume—V_0 and length—l_0 of the specimen were determined by the dimensions of the mold. The standard deviation of measurements on 15 specimens was found to be: $d(V_0) = 0.2$ cm^3 and $d(l_0) = 0.1$ cm.

The specimen is placed in the dilatometer after measuring its dimensions. The dilatometer is carefully filled with water, avoiding trapping of air bubbles, and the specimen is strained at a cross-head speed of 1 cm/min (the rate of extension $\dot{\epsilon} = 0.077$ min^{-1}). During straining the changes in the height of the water column in the capillary are recorded manually by means of a remotely controlled marker on the same chart of the Instron which records the force and the extension. In this way the dilatation, extension and stress are recorded simultaneously.

COMPUTATION OF POISSON'S RATIO AND ESTIMATION OF ERRORS

The volume change if the strained specimen ΔV is equal to the difference between the volume change measured by the height of the water column in the capillary tube and the volume of the section of the rod which is pulled out from the dilatometer's chamber while straining the specimen:

$$\Delta V = v_m h_m - v_c h_c \tag{16}$$

Substitution of relation (15) in Eq. (16) gives:

$$\Delta V = v_c \left[\frac{h_m}{A} - h_c \right] \tag{17}$$

Since we used specimens with metal terminal blocks, the extension of the specimen Δl equals to the length of the pulling rod withdrawn from the chamber—h_m. Knowing the extension and the volume change, Poisson's ratio can be calculated.

Expressing Poisson's ratio according to the various definitions of strain (Table I) and the dilatation (Eq. 17) as a function of the measured independent variables: v_c, h_m, h_c, V_0, l_0, the error in the value of Poisson's ratio can be estimated. The variation in Poisson's ratio due to the variation in these parameters, may be determined by evaluating the complete differential $d\nu$ in terms of the partial derivatives of the variables.[8]

$$|d\nu| \leq [|C_1|d(v_c) + |C_2|d(h_m) + |C_3|d(h_c) + |C_4|d(V_0) + |C_5|d(l_0)] \tag{18}$$

The values of the terms C_1 to C_5 differ according to the various definitions of Poisson's ratio and are given in Table II.

A computer program was coded for the calculation of Poisson's ratio according to its various definitions and the experimental error.

EXPERIMENTAL RESULTS AND DISCUSSION

The experiments were performed at a constant room temperature. An example of the experimental results is shown in Table III. For each material at least three specimens were tested. At each extension the volume change was measured, so that it was possible to show the variation of Poisson's ratio as a function of strain. The calculations of Poisson's ratio and the experimental error, were repeated according to the various definitions of strain at each measured point.

The variation of Poisson's ratio as a function of strain is shown in Figure 3, plotted so as to exaggerate very small differences. Hencky's logarithmic definition of strain (4) results in values of Poisson's ratio similar to the results obtained according to the elastic definition. The reason for it is obvious since for small dilatation $\ln[(V_0 + \Delta V)/V_0]$ is almost equal to $\Delta V/V_0$.

For elementary engineering calculations it is sufficient to adopt a constant value for Poisson's ratio, but it is known that for elastomers, in spite of the fact that their dilatation is very small, Poisson's ratio is strain and even time dependent.[1] The only definition of Poisson's ratio which describes such a behaviour is Cauchy's definition (10). In Figure 3 it can be seen how Poisson's ratio decreases as the strain increases.

TABLE II

The terms of the partial derivatives of the variables in Eq. (18) for the various definitions of strain

Term	Definition		
	Cauchy	Hencky	Engineering Hooke's law
C_1	$-\frac{\Delta V}{2\epsilon_x B^{1/2}\lambda_x^{1/2} V_0 v_c}$	$-\frac{\Delta V}{2 v_c V_0 B \ln \lambda_x}$	$-\frac{\Delta V}{\epsilon_x V_0 v_c}$
C_2	$\frac{B^{1/2}(2+3\epsilon_x)}{2\epsilon_x^2 l_0 \lambda_x^{3/2}} - \frac{v_c}{2\epsilon_x B^{1/2} V_0 A \lambda_x^{1/2}} - \frac{l_0}{h_m^2}$	$\frac{\ln B}{2 l_0 \lambda_x \ln \lambda_x} - \frac{v_c}{2 V_0 B A \ln \lambda_x}$	$\frac{\Delta V}{\epsilon_x^2 V_0 l_0} - \frac{v_c}{\epsilon_x V_0 A}$
C_3	$\frac{v_c}{2\epsilon_x B^{1/2}\lambda_x^{1/2} V_0}$	$\frac{V_c}{2 B V_0 \ln \lambda_x}$	$\frac{V_c}{\epsilon_x V_0}$
C_4	$\frac{\Delta V}{2\epsilon_x B^{1/2}\lambda_x^{1/2} V_0^2}$	$\frac{\Delta V}{2 B V_0^2 \ln \lambda_x}$	$\frac{\Delta V}{\epsilon_x V_0^2}$
C_5	$\frac{1}{h_m} - \frac{B^{1/2} h_m}{2\epsilon_x^2 l_0^2 \lambda_x^{3/2}}$	$-\frac{h_m \ln B}{\lambda_x l_0^2 (\ln \lambda_x)^2}$	$-\frac{\Delta V}{\epsilon_x V_0 l_0}$

where $B = \frac{V_0 + V}{V_0}$ and $\lambda_x = 1 + \epsilon_x$

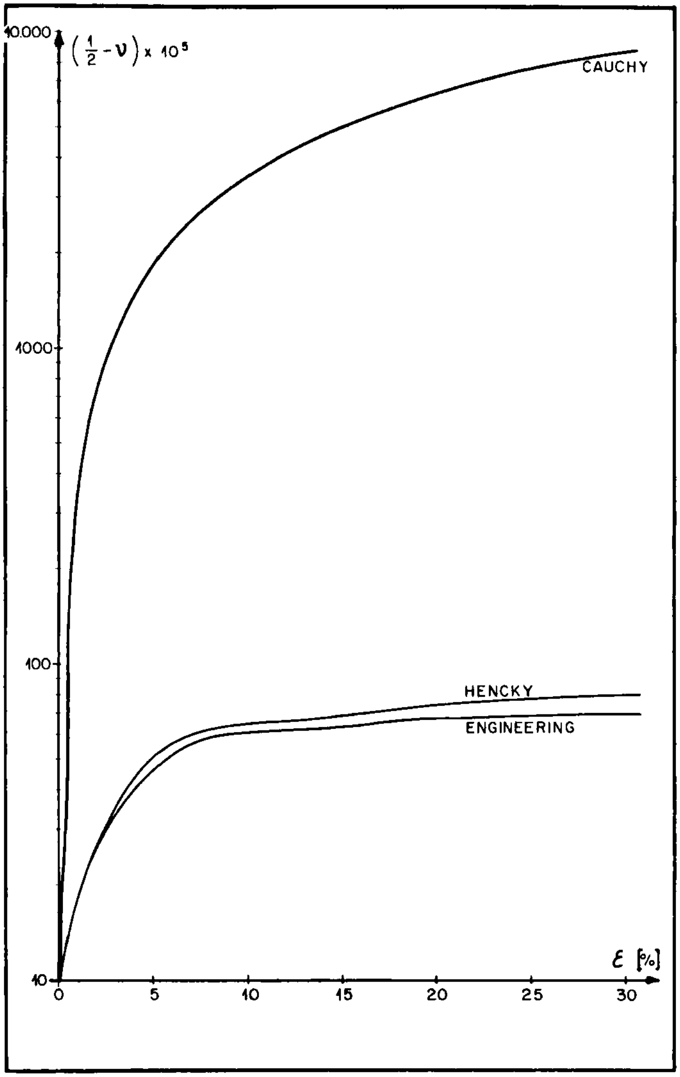

FIGURE 3 The variation of Poisson's ratio as a function of strain calculated according to three definitions of strain. (Note that the ordinates are drawn to exaggerate the minute changes of ν).

Another interesting result is that at low strains there is no significant difference between the values of Poisson's ratio of filled and unfilled elastomers. Only after a certain strain has been reached ($\epsilon = 14\%$) the dewetting process, occurring between the filler particles and the binder[2] cause a faster increase of dilatation (see Figure 4). The last change in the rate of dilatation noticed in the filled system ($\epsilon \simeq 30\%$) is due to the beginning of rupture processes in the specimen. In Figure 5 the effect of rate of extension is shown. As the rate is increased, the dilatation is first lower, then higher and of course, the Poisson's ratio decreases.

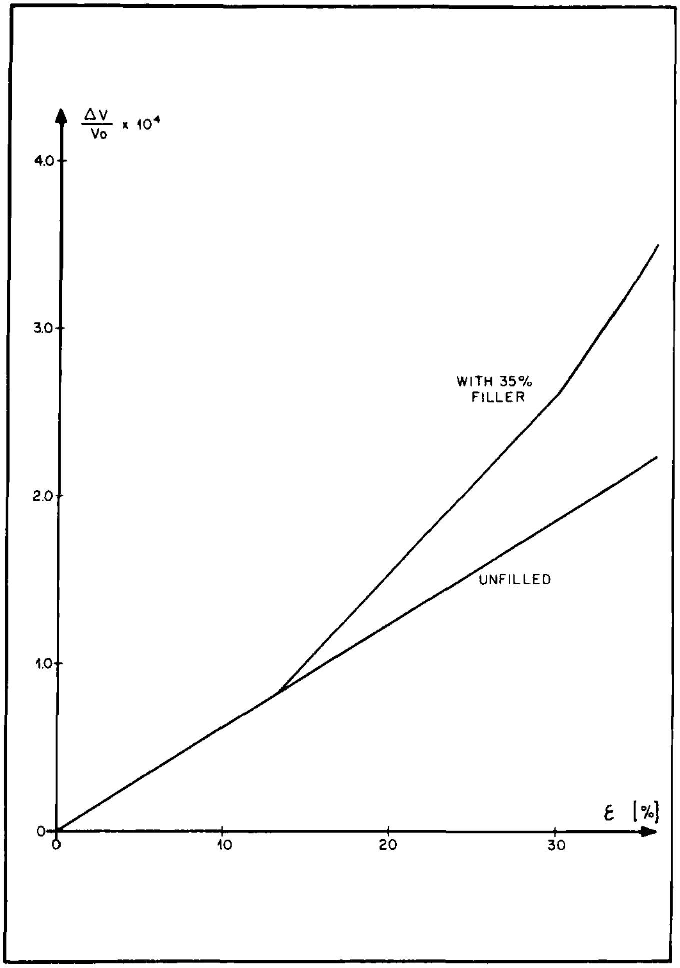

FIGURE 4 Dilatation of an unfilled and filled elastomer.

CONCLUSIONS

A simple and precise dilatometer to fit a universal Instron testing machine was built. It made it possible to measure Poisson's ratio of nearly incompressible

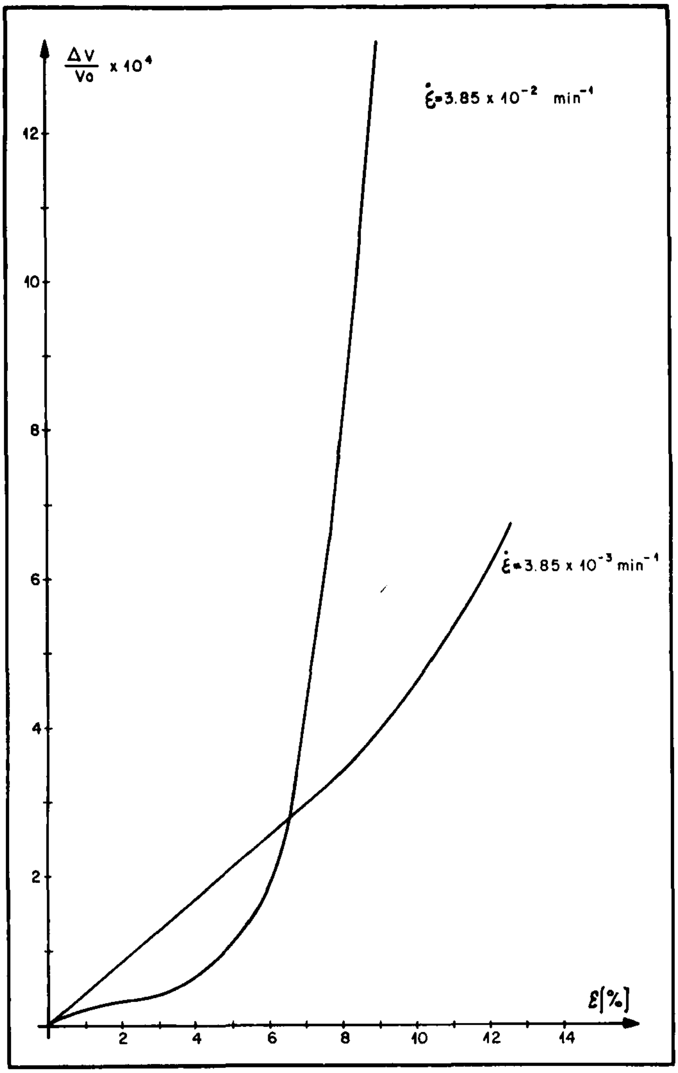

FIGURE 5 Effect of rate of strain on dilatation.

elastomers as a function of strain and rate of strain within an accuracy of ± 0.0002 (about 0.05 %). This apparatus can be used also for the determination of the stress–strain behaviour of elastomers as a function of dilatation; in this way the effect of dewetting can be studied and the bulk modulus can be calculated.

Work is in hand to modify the apparatus so as to carry out measurements at non-ambient temperatures.

TABLE III

Example of a measurement of an unfilled elastomer (cf. Figure 3)

No.	ΔV	Strain	Poisson's R. Engineering	Error P.R. Engineering	Poisson's R. Cauchy	Error P.R. Cauchy	Poisson's R. Hencky	Error P.R. Hencky
1	.00077	.0085	.49948	.00039	.49633	.00061	.49947	.00039
2	.00136	.0262	.49970	.00015	.49010	.00036	.49970	.00015
3	.00298	.0385	.49955	.00012	.48558	.00033	.49954	.00013
4	00461	.0506	.49948	.00011	.48122	.00032	.49946	.00011
5	.00505	.0600	.49951	.00010	.47810	.00030	.49950	.00010
6	.00756	.0746	.49941	.00010	.47309	.00030	.49939	.00010
7	.01007	.0892	.49935	.00010	.46822	.00029	.49932	.00010
8	.01140	.1008	.49935	.00010	.46451	.00028	.49931	.00010
9	.01184	.1100	.49938	.00009	.46161	.00028	.49934	.00010
10	.01317	.1215	.49937	.00009	.45801	.00027	.49934	.00009
11	.01420	.1323	.49938	.00009	.45471	.00026	.49934	.00009
12	.01582	.1448	.49937	.00009	.45098	.00026	.49932	.00009
13	.01745	.1569	.49936	.00009	.44733	.00026	.49931	.00009
14	.01907	.1692	.49935	.00009	.44374	.00025	.49929	.00009
15	.02158	.1838	.49932	.00009	.43954	.00025	.49926	.00009
16	.02380	.1977	.49930	.00009	.43565	.00025	.49924	.00010
17	.02424	.2069	.49932	.00009	.43312	.00024	.49926	.00009
18	.02557	.2185	.49932	.00008	.42999	.00024	.49925	.00009
19	.02778	.2323	.49931	.00009	.42629	.00023	.49923	.00009
20	.02911	.2438	.49931	.00008	.42327	.00023	.49923	.00009
21	.03132	.2577	.49930	.00009	.41969	.00023	.49921	.00010
22	.03236	.2685	.49930	.00808	.41697	.00023	.49921	.00009
23	.03369	.2890	.49930	.00006	.41409	.00022	.49921	.00009
24	.03531	.2923	.49930	.00008	.41106	.00022	.49920	.00010
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